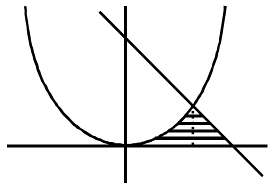
EXERCISE – III

HINTS & SOLUTIONS

Sol.1 $A = \int_0^1 x^2\,dx + \frac{1}{2} = \frac{5}{6}$



Sol.2 $\left|\int_{\pi/6}^{c} \sin 2x\,dx\right| = \frac{1}{2}$

on solving $c = -\frac{\pi}{6}$ on $\frac{\pi}{3}$

Sol.3 Equation of tangent at (x_0, x_0^2)

$2x\,x_0 - y = x_0^2$ ....(i)

line : $y = x_0^2$ ....(ii)

axis of ordinates y-axis ....(iii)

on solving (i) & (ii) & (i) & (iii)

area of triangle

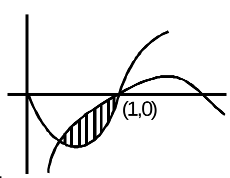
$A = x_0^3$

It will be greatest for $x_0 = 2$

so $A = 8$

Sol.4 Intersecting points are $\frac{1}{e}$ & 1

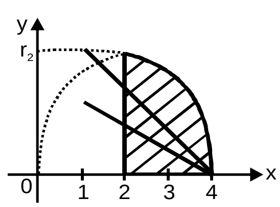
so $A = \int_{1/e}^{1}\left(\frac{\ell n\,x}{ex} - ex\,\ell n\,x\right)dx$



$= \frac{e^2 - 5}{4e}$ sq. units.

Sol.5 $A = \left|\int_2^4 \sqrt{2}\sin\frac{\pi x}{4}\,dx\right|$

$= \frac{4\sqrt{2}}{\pi}$ sq. units.

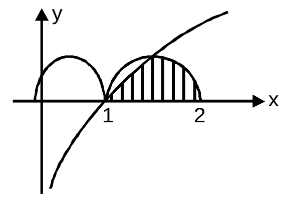


let the line is y=mx–4m

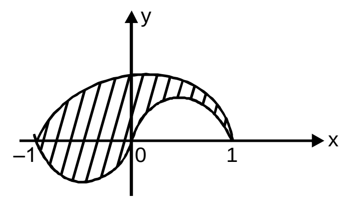
$A = \left(\frac{4\sqrt{2}}{\pi}\right) \times \frac{1}{3} = \int_2^4\left(\sqrt{2}\sin\frac{\pi x}{4} - mx + 4m\right)dx$

similarly for m_2

Sol.6



Sol.7



$A = \left|\int_{-1}^{1}\left(\sqrt{1-x^2} - x^3 + x\right)dx\right|$

Sol.8 $A = 2\int_0^{\sqrt{a/2}}(2x^2 - a)\,dx = 18\sqrt{2}$

$\Rightarrow a^{3/2} = 9 \times 3 \Rightarrow a = 9$

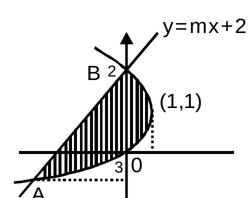
Sol.9 point of intersection of line & curve = (3, 2) & (1, 5).

Sol.10 Intersecting points = 0, ± 2

so area $= 2\int_0^2 (2x^2 - x^4 + 2x^2)\,dx$

$= \frac{128}{15}$

Sol.11 (a) Area$= \frac{1}{2}.2 \times \left(\frac{-1-2m}{m^2}\right) + \int_0^1 (2y - y^2)\,dy = \frac{3}{2}$

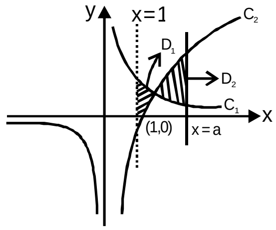


(b) If area is minimum $\frac{dA}{dm} = 0$

$\Rightarrow m = \infty$

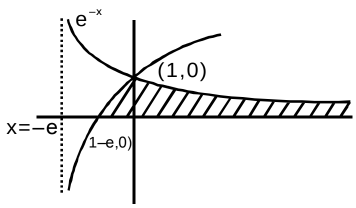
& $A_{min} = \frac{4}{3}$.

Sol.12



Sol.13 x = log e (1/y)

$\Rightarrow$ $y = e^{-x}$



$$A = \int_{1-e}^{0} \log(x+c)\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

$= 2$

Sol.14 Intersecting points of the curve is

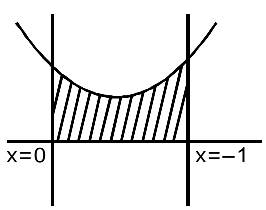
$x = -a$ & $x = -2a$

$$\text{so } \; x = \int_{-2a}^{-a} \left(\frac{(a^2 - ax)}{1 + a^4} - \left(\frac{x^2 + 2ax + 3a^2}{1 + a^4} \right) \right) dx$$

$$\frac{dA}{da} = 0 \qquad \Rightarrow \; \underbrace{a = 3^{1/4}}$$

Sol.15 $a^2x^2 + ax + 1$ is clearly positive

$$A = \int_0^1 (a^2x^2 + ax + 1)\,dx$$



$$= \frac{1}{6}\left(2\left(a + \frac{3}{4}\right)^2 + \frac{39}{8}\right)$$

minimum for $a = -3/4$.

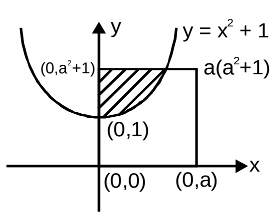
Sol.16 Area of rectange = $a \cdot (a^2 + 1)$

now area of shaded region.

$$\text{now } \int_1^{a^2+1} \sqrt{y-1}\,dy$$

$$= \frac{1}{2}\,[a \cdot (a^2 + 1)]$$

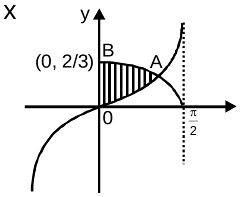
$\Rightarrow$ $a = \sqrt{3}$



Sol.17 on solving $\tan x = \frac{2}{3}\cos x$

$3 \tan x = 2 \cos x$

$\Rightarrow$ $x = \frac{\pi}{6}$



$$\text{so area is } = \int_0^{\pi/6} (2/9 \cos x - \tan x)\,dx$$

Sol.18 $y = xe^{-x}$

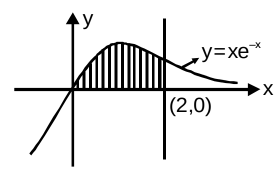
$$\frac{dy}{dx} = -xe^{-x} + e^{-x} = e^{-x}(1-x)$$

$$\frac{d^2y}{dx^2} = e^{-x}(-1) + (1-x)\cdot(-e^{-x})$$

$= e^{-x}\,[x - x) = 0 \;\Rightarrow\; x = 2 \quad \{\because\; e^{-x} \neq 0\}$

so c = 2

$$A = \int_0^2 (xe^{-x})\,dx$$



$$= -\,xe^{-x}\Big|_0^2 + \int_0^2 e^{-x}\,dx$$

$= 1 - 3e^{-2}$ sq. units

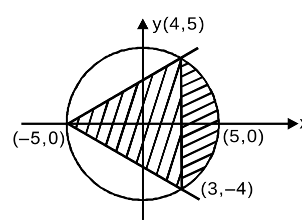
Sol.19 $\left| \int_1^e (8x^2 - x^5)\,dx \right| = \frac{16}{3}$

Sol.20

Total area

= area of Δ

+ area of sector



Sol.21 $y = x\overline{e}^{x^2}$

$$y' = \overline{e}^{x^2} - 2x^2\,e^{-x^2} = 0 \;\Rightarrow\; x = \pm\frac{1}{\sqrt{2}}$$

y is maximum at $x = \frac{1}{\sqrt{2}}$

$$\text{Area} = \int_0^{1/\sqrt{2}} x\,\overline{e}^{x^2}dx = \frac{1}{2}\,(1 - e^{-1/2})$$